Same instructions as Mission 1. Thanks!

Problem 161 Let f be a smooth function and $\vec{G}$ a smooth vector field on some subset of $\mathbb{R}^3$. Show that $\nabla \bullet (f\vec{G}) = \nabla f \bullet \vec{G} + f\nabla \bullet \vec{G}$ and $\nabla \times (f\vec{G}) = \nabla f \times \vec{G} + f\nabla \times \vec{G}$

Problem 162 Let f be a smooth function and $\vec{G}$ a smooth vector field. Show that

$$\nabla \times (f\vec{G}) = \nabla f \times \vec{G} + f\nabla \times \vec{G}$$

Problem 163 Let $f(x, y, z) = x^2 + yz$ and $\vec{G}(x, y, z) = \langle x, z, z^2\rangle$. Let $\vec{H} = f\vec{G}$ and calculate the curl and divergence of $\vec{H}$.

Problem 164 Let k be a constant and

$$\vec{F}(x, y, z) = \frac{k}{(x^2 + y^2 + z^2)^{3/2}}\langle x, y, z\rangle.$$

Calculate the curl and divergence of $\vec{F}$.

Problem 165 Let $\vec{B}(x, y, z) = \langle -y, x + z, 2\rangle$. Find $\vec{A}$ for which $\nabla \times \vec{A} = \vec{B}$. To my grader: sorry about this. There are infinitely many correct answers.

Problem 166 Let C be the curve with parametrization $x = e^t \cos(t)$, $y = e^t \sin(t)$ and $z = e^t$ for $0 \leq t \leq \frac{1}{2}\ln 3$. Assume the mass-density of C is uniform and find the center of mass of the wire as well as its arclength.

Problem 167 Let C be the curve with parametrization $x = 2 + 3t^2$ and $y = t^3 - 1$ for $0 \leq t \leq 1$. If $\vec{F}(x, y) = \langle 3x^2 + y, 3y^2 + x\rangle$ then calculate $\int_C \vec{F} \bullet d\vec{r}$.

Problem 168 Calculate $\int_L y^2 dx + x^3 dy + z dz$ where L is the line-segment from $(1, 2, 3)$ to $(4, 2, 1)$.

Problem 169 Let $\vec{F} = \langle\ y,\ x,\ 2z\ \rangle$. Let C be a path from $(7, 8, 1)$ to $(-2, 5, 0)$. Calculate $\int_C \vec{F} \bullet d\vec{r}$.

Problem 170 Let $\vec{F}(x, y) = \langle x^3 + \sin(y), y^3 + \tan(x)\rangle$. Calculate the flux of $\vec{F}$ through the unit-circle $x^2 + y^2 = 1$. I recommend you use the divergence form of Green's Theorem for this problem. Direct calculation of the flux seems less fun.

Problem 171 Let R be the rectangle with corners $(0, 0), (1, 0), (1, 3)$ and $(0, 3)$. Let ∂R be the CCW oriented boundary of R as the notation ∂R indicates. Calculate by Green's Theorem:

$$\oint_{\partial R} \big(\cosh(\sqrt{x} + 3) + y\big)\, dx + \big(3 + x^2\big)\, dy.$$

Problem 172 Let C be a path from $(0, 1)$ to $(2, 3)$. Calculate $\int_C (1 + 4x^3)dx + (2 - 2y^2)dy$.

Problem 173 Find the work done by the force $\vec{F}(x,y,z) = \langle y+x^2, x, z^3\rangle$ on a mass as it moves from $(0,0,0)$ to $(1,2,3)$.

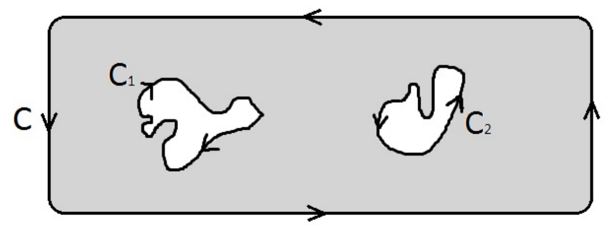
Problem 174 Determine if the vector fields below are conservative. Find potential functions where possible.

(a.) $\vec{F}(x,y) = \langle 2x - \sin(x+y^2), -2y\sin(x+y^2)\rangle$

(b.) $\vec{F}(x,y) = \langle -y, x\rangle$

(c.) $\vec{F}(y,z) = \langle z+y^2, y+z^3\rangle$

Problem 175 Suppose we are given $\displaystyle\int_{C_1} \vec{F} \bullet d\vec{r} = -10$ and $\displaystyle\int_{C_2} \vec{F} \bullet d\vec{r} = 9$. Calculate $\displaystyle\int_{C} \vec{F} \bullet d\vec{r}$ given that $\vec{F}$ is conservative (locally) on the domain between C and the curves C_1, C_2. Is $\vec{F}$ conservative on $\mathbb{R}^2$? Explain.



Problem 176 Let $\vec{F} = \langle 0, 0, -mg\rangle$ where m, g are positive constants and suppose $\vec{F}_f = -b\vec{T}$ where v is your speed and b is a constant and $\vec{T}$ is the unit-vector which points along the tangential direction of the path. This is a simple model of the force of kinetic friction, it just acts opposite your motion. Find the work done by $\vec{F}_f + \vec{F}$ as you travel up the helix $\vec{r}(t) = \langle R\cos(t), R\sin(t), t\rangle$ for $0 \leq t \leq 4\pi$.

Problem 177 Let C be the CCW square with vertices $(0,0), (1,0), (0,1), (1,1)$. Calculate

$$\int_C (x-y^3)dx + (\sin y + x^4)dy$$

Problem 178 What are the four equivalent characterizations of a conservative vector field over a simply connected subset of the plane ? If we replace a simply connected subset of the plane with a set which is merely connected then which of the four conditions is no longer equivalent. Can you give an example which illustrates why the fourth condition is lost ? Finally, which three of these continue to be equivalent characterizations of a conservative vector field in the context of a connected subset of $\mathbb{R}^n$?

Problem 179 Complete the proof of Green's Theorem. We did half in class, do the other half here.

Problem 180 Show Green's Theorem with holes follows from Green's Theorem for a simply connected subset of the plane.