MATH 231	QUIZ 4

Please work the problems in the white space provided and clearly box your solutions. You are allowed one $3'' \times 5''$ *notecard. Enjoy!.*

Problem 1 (10pt) Suppose C_1 is a curve parametrized by $x = t, y = t^2$ and $z = t^3$ for $0 \leq t \leq 1$. Calculate the line integral $\int_{C_1} z dx + y^2 dy + x^3 dz$.

Problem 2 (5pt) Suppose C_2 is another path from $(0, 0, 0)$ to $(1, 1, 1)$. What can we say about $\int_{C_2} z dx + y^2 dy + x^3 dz$. Will the value be the same as in the previous problem? Describe the possibilities.

Problem 3 (5pt) Is $\vec{F}(x, y, z) = \langle e^x,\ ye^{y^2},\ 3z^2 \rangle$ a conservative vector field on $\mathbb{R}^3$? Explain why it is, or why it is not.

Problem 4 (16pt) Suppose $\vec{F}(x,y,z) = \langle 2xy^2,\ 2x^2y + 2y + 2z,\ y^2 \rangle$

(a.) calculate $\nabla \bullet \vec{F}$

(b.) calculate $\nabla \times \vec{F}$

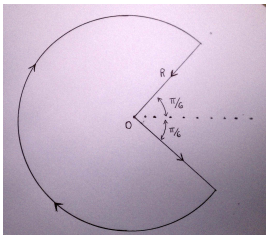
(c.) if possible find f such that $\vec{F} = \nabla f$.

(d.) let C be the curve parametrized by $x = 3+2t, y = 2t^2$ and $z = 4t^3$ for $0 \leq t \leq 1$. Calculate $\int_C \vec{F} \bullet d\vec{r}$

Problem 5 (6pt)Let C be a circle of radius R centered at $(0,0)$. Let $\vec{F}(x,y) = \langle x^3, y^3 \rangle$. Calculate the flux of $\vec{F}$ through C.

Problem 6 (10pt) Let C be the curve illustrated below which is formed by part of a circle of radius R centered at the origin and two line-segments at $\theta = \pm\pi/6$. Calculate

$$\int_C x^2 y\, dx - y^2 x\, dy.$$



Problem 7 (10pt) Suppose $\vec{F}(x,y,z) = \langle z,\ \cosh^2(x+y+z),\ x^3 \rangle$ and suppose S is the surface parametrized by

$$\vec{r}(u,v) = \langle u,\ 3+v\ , 2-u^2 \rangle$$

for $(u,v) \in [0,1] \times [0,1]$. Calculate the flux of $\vec{F}$ through S.

Problem 8 (11pt) Find the flux of $\vec{F}(x,y,z) = \langle xy^2, y^3, zy^2 \rangle$ through the surface of the positively oriented cube $E = [0,1]^3$. That is calculate $\int_{\partial E} \vec{F} \cdot d\vec{S}$.

Problem 9 (15pt) Find the flux of the vector field $\vec{F} = \langle x, y, z \rangle$ through the boundary of the simple solid E which is bounded by $x^2 + y^2 = 1$ and $z = x$ and $z = 4 - y$.

Problem 10 (10pt) Calculate $\int_S \left(\nabla \times \vec{F} \right) \bullet d\vec{S}$ for $\vec{F}(x, y, z) = \langle zx, zy, z^2 \rangle$ over the positively oriented hemisphere $\rho = R$ with $z \geq 0$. Hint: use Stokes' theorem to trade the hemisphere for an easier surface.

Problem 11 (6pt) Show that $\nabla \bullet (\nabla \times \vec{G}) = 0$.

Problem 12 (6pt) Consider $\vec{F}(x,y) = \frac{1}{\sqrt{x^2+y^2}}\langle -y, x\rangle$. Calculate the line-integral of $\vec{F}$ around the curve C parametrized by $x = 4\cos(t) + \sin(2t^2/\pi)$ and $y = 5\sin(t)$ for $0 \leq t \leq 2\pi$. I provide a plot of C below (the dotted curve is the unit-circle)

