MATH 231	TEST 4

Please work the problems in the white space provided and clearly box your solutions. You are allowed one $3'' \times 5''$ *notecard. Enjoy!.*

Problem 1 (10pt) Let C be a curve from $(1, 2, 3)$ to $(5, 5, 5)$ calculate:

$$\int_C xy^2 dx + yx^2 dy + dz.$$

Problem 2 (10pt) Find the surface area of the snow-cone-like shape formed by topping off the cone $\phi = \pi/4$ by a portion of a sphere of radius R.

Problem 3 (10pt) (this problem intends use of divergence theorem) Calculate the flux of $\vec{F}(x, y, z) = \langle x^3,\ -yx^2,\ -zx^2 \rangle$ through the positively oriented sphere S_R: $x^2 + y^2 + z^2 = R^2$. That is, calculate $\iint_{S_R} \vec{F} \bullet d\vec{S}$.

Problem 4 (10pt) Let Q be the rectangle formed by $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 1)$ and $(1, 1, 1)$ oriented CCW as viewed from above (from the positive z-axis). Use Stokes' Theorem to calculate $\int_Q x^2 dx + y^2 dy - x dz$.

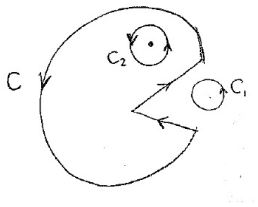
Problem 5 (10pt) Calculate the surface integral $\iint_S \vec{F} \cdot d\vec{S}$ for S where S is the subset of the plane $x+y+z=1$ bounded by the cylinder $x^2+y^2=1$ and $\vec{F}(x,y,z) = \langle 1,\ x^3,\ 0 \rangle$

Problem 6 (10pt) Suppose $\vec{F}(x,y,z) = \left\langle e^y - x,\ z^3,\ y^2 + z \right\rangle$

(a.) calculate $\nabla \times \vec{F}$

(b.) let C be the line segment from $(0,0,0)$ to $(1,2,3)$ for $0 \leq t \leq 1$. Calculate $\int_C \vec{F} \bullet d\vec{r}$.

Problem 7 (5pt) Suppose that $\vec{F} = \langle P, Q \rangle$ has $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ at all points except the at the centers of the CCW oriented circles C_1, C_2.It is given that $\int_{C_1} \vec{F} \bullet d\vec{r} = 1$ and $\int_{C_2} \vec{F} \bullet d\vec{r} = 2$. Find $\int_C P dx + Q dy$ where C is pictured below:

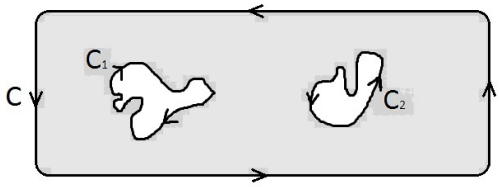


Problem 8 (10pt) **Let C be the CCW triangle with vertices $(-1, 0)$, $(1, 0)$ and $(0, 1)$. Calculate**

$$\int_C (xe^x - y^3)dx + (y^3 \cos(y) + x)dy$$

Problem 9 (5pt) Show that $\nabla \times \nabla f = 0$.

Problem 10 (5pt) Suppose we are given $\int_{C_1} \vec{F} \bullet d\vec{r} = 4$ and $\int_{C_2} \vec{F} \bullet d\vec{r} = 11$. Calculate $\int_{C} \vec{F} \bullet d\vec{r}$ given that $\vec{F}$ is conservative (locally) on the domain between C and the curves C_1, C_2. Is $\vec{F}$ conservative on $\mathbb{R}^2$? Explain.



Problem 11 (5pt) Show that if f solves Laplaces equation $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$ on a simply connected region R then the flux of ∇f through the boundary ∂R is zero.

Problem 12 (10pt) Set-up, but do not evaluate, the surface-integral of $\vec{F}(x, y, z) = \langle 1, 2, \ x+y+z \rangle$ on the positively oriented ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$. (I want you to state the parameterization you would use and work out the normal vector field)

Problem 13 (10pt) Continuing the previous problem, use the divergence theorem to calculate the flux of $\vec{F}$ through the ellipsoid.