This sheet must be printed and attached to your assignment as a cover sheet. The calculations and answers should be **written** (not printed from a tablet) neatly on one side of paper which is attached and neatly stapled in the upper left corner. Box your answers where appropriate. Please do not fold. Thanks! Typically each problem is worth 2pts, so I have included some bonus to earn, for this reason I will be picky about missed steps in certain places, I need to see the logical reasoning and algebra steps. For proofs, you need to work with the definitions given and model the work I do in lecture as well as use theorems as appropriate. For example, Calculus III is a prerequisite for this course, so you can cite Green's Theorem or the Fundamental Theorem of line integrals. We also assume the theory of continuity of elementary functions from Calculus III so I don't expect you to prove such things, but you should claim such things when it is time and critical to your logical claim (looking ahead to Mission 3 really).

Please note this Mission, as well as the later ones, are based in part on my Course Planner which indicates the sections in my Lecture Notes which are most relevant. I hope you look at my notes. I will be rather disappointed if I see notation I never introduced used in your homework solutions. I am here to help, you just have to ask. You can ask questions in office hours, but do keep in mind that my office is not the place to work out the homework.

Finally, this is a 300-level math major course. There will be homework problems which are not just mirrors of class examples. You must think and extrapolate from what we do in class to extend the concepts to the problems you face here. Some of them are very difficult, but if you get through them you will be forever changed. Like a Saiyan recovering from a mortal wound, I wish you to rise from the ashes and destroy my next Boss Fight.

Problem 1 Let $z = 1 + i$ and $w = 3 - 3i$. Find the Cartesian and polar forms of

- **(a.)** z^8
- **(b.)** zw
- **(c.)** $\dfrac{z}{w}$
- **(d.)** $\dfrac{3i}{w} + \dfrac{1}{z}$

Problem 2 Let $z, w \in \mathbb{C}$. Show that $\overline{zw} = \overline{z}\,\overline{w}$.

Problem 3 Suppose z, w are nonzero complex numbers. Show $|z+w| = |z|+|w|$ if and only if $z = kw$ for some nonzero positive real number k.

- **(a.)** Use the law of cosines and a geometric argument to establish the claim,
- **(b.)** establish the claim via explicit arguments based on algebra and the polar decomposition of complex numbers.

Problem 4 Calculate $(-2i)^{1/5}$ and $\sqrt[5]{-2i}$.

Problem 5 Let b, c be complex numbers. Derive the solution set of $z^2 + bz + c = 0$. Your solution should be given in terms of the principal square root as appropriate.
This result is not known from your previous course, it is a happy accident that the formula still works with the proper interpretation.

Problem 6 Find all solutions of $z^2 + 2z + (1 - i) = 0$.

Problem 7 Find all solutions of $\sqrt{2}z^4 = 81(1 + i)$. Also, factor $f(z) = z^4 - 81e^{i\pi/4}$.

Problem 8 Find all solutions of $e^{z-i} = i$.

Problem 9 Calculate $Log(1 + i)$ and $\log(1 + i)$.

Problem 10 Sketch the following sets in the complex plane:

(a.) $\{z \in \mathbb{C} \mid |z - 3 - 4i| \leq 5\}$
(b.) $\{z \in \mathbb{C} \mid |z - i| + |z + i| = 3\}$
(c.) $\{z \in \mathbb{C} \mid |z| = |z + 1|\}$
(d.) $\{z \in \mathbb{C} \mid e^z = i\}$

Problem 11 Let $z, w \in \mathbb{C}^\times$. Let $p \in \mathbb{C}$, we define the set of values $z^p = exp(p \log(z))$. Show that:

(a.) $Arg(z) + Arg(w) = Arg(zw) + 2\pi n$ for some $n \in \mathbb{Z}$,
(b.) $Log(z) + Log(w) = Log(zw) + 2\pi ni$ for some $n \in \mathbb{Z}$,
(c.) $z^p w^p = (zw)^p$ (*this is a statement about sets generally*).

Problem 12 Show $Log(e^z) = z$ if and only if $-\pi < Im(z) \leq \pi$.