Same instructions as Mission 1. Thanks !

Problem 25 Let $f(z) = (iz+3)^2$. In each case, prove $f'(z)$ exists and calculate its formula:

(a.) via the difference quotient definition,

(b.) via the Caratheodory formulation,

(c.) via the Cauchy Riemann Theory formulation,

(d.) via the Wirtinger derivatives formulation.

Problem 26 Let $f(z) = \frac{1}{z^2+1}$.

(a.) Show $f'(z) = \dfrac{-2z}{(z^2+1)^2}$ via the difference quotient definition,

(b.) Show $f'(z) = \dfrac{-2z}{(z^2+1)^2}$ via the Caratheodory formulation,

(c.) Show $f'(z) = \dfrac{-2z}{(z^2+1)^2}$ via the Cauchy Riemann Theory formulation,

(d.) Show $f'(z) = \dfrac{-2z}{(z^2+1)^2}$ via the Wirtinger derivatives formulation.

Problem 27 Let $f(z)$ be a branch cut of z^c for some complex power c. Show $\frac{d}{dz}z^c = cz^{c-1}$. Also, show $\frac{d}{dz}\sqrt{z} = \frac{1}{2\sqrt{z}}$.

Problem 28 Let $f(z) = Log(z^2+1)$. If $f = u+iv$ then show u, v satisfy the Cauchy Riemann equations. Verify $f'(z) = \frac{\partial f}{\partial x}$.

Problem 29 Calculate $\frac{d}{dz}\tan^{-1}(z)$.

Problem 30 Let $f(x+iy) = x^3y^2 + ix^2y^3$. Find all points at which f is complex differentiable. Also, find any points where f is **holomorphic**.

Problem 31 Let $f(z) = \cos z = \frac{1}{2}(e^{iz} + e^{-iz})$.

(a.) Show $\frac{d}{dz}\cos z = -\sin z$ using the chain-rule,

(b.) Show $\frac{d}{dz}\cos z = -\sin z$ via the Cauchy Riemann Theory.

Problem 32 In the previous Mission we proved $\sin(z+w) = \sin z \cos w + \cos z \sin w$. Fix w and differentiate with respect to z to find the adding angles formula for $\cos(z+w)$.

Problem 33 Calculate the limit using complex calculus: $\displaystyle\lim_{z\to i} \frac{\sinh z + i\sin(1)}{z-i}$.
Please note: L'Hopital's Rule is not an option here, thanks!

Problem 34 Suppose $\lim_{z\to a} f(z)$ and $\lim_{z\to a} g(z)$ both exist and let c be a nonzero complex number. Prove the limit law $\lim_{z\to a}[cf(z)+g(z)] = c\lim_{z\to a} f(z) + \lim_{z\to a} g(z)$ using the $\varepsilon - \delta$ definition of the limit.

Problem 35 Prove the complex derivative rules using the difference quotient definition of complex differentiability. You may assume the limit laws for the complex limit are known. Suppose f and g are differentiable at $z = a$.

(a.) Show $(f + g)'(a) = f'(a) + g'(a)$.

(b.) Show $(fg)'(a) = f'(a)g(a) + f(a)g'(a)$.

Problem 36 Let f be complex differentiable at $z = z_o$. Prove f is continuous at $z = z_o$.