Same instructions as Mission 1. Thanks !

**Problem 49** Given that $\mathbb{R}$ is complete, show $\mathbb{C}$ is complete.

**Problem 50** Show, for $k \geq 1$, $f_k(x) = \dfrac{x^k}{k + x^{2k}}$ converges uniformly on $[0, \infty)$. *Hint: use calculus to determine the worst-case estimator*

**Problem 51** Find the radius of convergence for each of the following power series:

**(a.)** $\displaystyle\sum_{n=0}^{\infty} \frac{z^n}{e^n}$,

**(b.)** $\displaystyle\sum_{n=1}^{\infty} n! \frac{z^n}{n^n}$,

**(c.)** The Taylor series generated by $f(z) = \dfrac{1}{z^2 - 3z + 2}$ at $z_0 = 2 + 3i$.

**Problem 52** Show that $\displaystyle\sum_{k=1}^{\infty} \frac{z^k}{k^2}$ converges uniformly for $|z| < 1$.

**Problem 53** Prove that an entire even function has an even power series.

**Problem 54** Let $f(z) = \displaystyle\sum_{n=0}^{\infty} \frac{z^n}{n!}$. Prove $f(z)f(w) = f(z + w)$ via series arguments.

**Problem 55** Calculate the power series expansion centered at $z = 0$ up to order 5 for the following functions:

**(a.)** $f(z) = \dfrac{z}{\sin z}$

**(b.)** $g(z) = \dfrac{e^z}{1 + z}$

**Problem 56** Expand the following functions about $\infty$ and write the first three nontrivial terms in the expansion ( I want both the $\Sigma$-notation form of the expansion and the explicit form of the first three nontrivial terms )

**(a.)** $f(z) = \dfrac{1}{z^2 + 1}$

**(b.)** $f(z) = \dfrac{2z}{3 + z}$

**(c.)** $f(z) = \cos(3 + 1/z)$

**Problem 57** Suppose $f(z) = z^3 \sin(z^2)$. Calculate $f^{(n)}(i)$. Break into cases as needed.

**Problem 58** For the functions below, find each zero of the function as well as the order of the zero.

**(a.)** $f(z) = \dfrac{z^2+4}{z^2-4}$

**(b.)** $f(z) = \cos(z) - 1$

**(c.)** $f(z) = \dfrac{1}{z^2}(\cos(z) - 1) = -\dfrac{1}{2} + \dfrac{1}{4!}z^2 + \cdots$ (for $z \neq 0$, and $f(0) = -1/2$)

**Problem 59** Find each zero and its order for $f(z) = z^3 \sin(2z)$.

**Problem 60** Find the power series expansion of $Log(z)$ about the point $z_0 = i - 2$. Show the radius of convergence $R = \sqrt{5}$. Explain why this does not contradict the discontinuity at $z = -2$ of $Log(z)$. Let me ask a leading question: on what part of the disk of convergence is the power series equal to $Log(z)$ ? Furthermore, on the part of the disk where it is not equal to $Log(z)$, what function does the power series represent ?