

Same instructions as Mission 1. Thanks !

Problem 73 Calculate $\text{Res}[(41 + 6z^2) \sinh(1/z), 0]$

Problem 74 Find the partial fractions decomposition of $f(z) = \frac{z^2+1}{z(z^2-1)}$. Furthermore, without additional calculation, read-off the residues of $f(z)$ at each of its isolated singular points.

Problem 75 Calculate the residue of $\frac{\cos z}{z^3}$ at $z = 0$.

Problem 76 Calculate the residue of $f(z) = \tan(\pi z)$ at each isolated singularity of $f(z)$ in the complex plane.

Problem 77 Calculate $\oint_{|z|=2} \frac{z}{\cos z} dz$.

Problem 78 Calculate $\oint_{|z-1|=1} \frac{1}{z^8 - 1} dz$.

Problem 79 Calculate $\int_{|z|=9} \left(\frac{1}{z-i} + \frac{1}{z-10} \right) dz$. Hint: the integrand is clearly a closed form at all points in $\mathbb{C}$ except $z = i$ and $z = 10$.

Problem 80 Calculate $\int_{|z|=9} \frac{\sin(z^2)}{(z - \pi i)^3} dz$.

Problem 81 Show via residue theory that $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2 + 1)^2} = \frac{\pi}{2}$

Problem 82 Assume $a > b > 0$. Show via residue theory that $\int_0^{2\pi} \frac{d\theta}{a + b \sin \theta} = \frac{2\pi}{\sqrt{a^2 - b^2}}$.

Problem 83 Assume $0 < a < 1$. Show that $\int_0^{\infty} \frac{x^{-a} dx}{1+x} = \frac{\pi}{\sin \pi a}$. Use the keyhole contour and appropriate techniques of residue calculus.

Problem 84 Show via residue theory that $\int_{-\infty}^{\infty} \frac{dx}{(x^2 + a^2)^2} = \frac{\pi}{2a^3}$ given that $a > 0$.