Same instructions as Mission 1. Thanks !

**Problem 97** Find a fractional linear transformation $T$ which:

**(a.)** carries the circle $|z| = 1$ onto the line $Re((1+i)w) = 0$,

**(b.)** carries the circle $|z| = 1$ onto the circle $|w-1| = 1$,

**(c.)** maps the circle $|z - z_o| = r$ onto the circle $|w| = 1$,

**Problem 98** Find a fractional linear transformation $T$ which is subject the following conditions:

**(a.)** $T(0) = 0$, $T(1) = 1$ and $T(i) = \infty$,

**(b.)** $T$ maps the real axis to itself and the imaginary axis onto the circle $\left|w - \frac{5}{4}\right| = \frac{3}{4}$

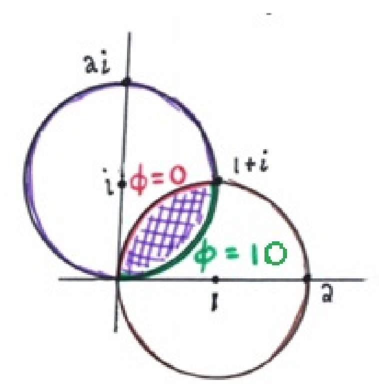
**(c.)** $T$ maps the real axis to itself and the line $y = x$ onto the circle $|w + i| = \sqrt{2}$.

**Problem 99** Find a solution to Laplace's equation on the annulus bounded by a radius 1 circle $C_1$ containing $1, i$ and a radius 2 circle $C_2$ containing $1-i, -1+i$. In particular, solve the boundary value problem $\phi_{xx} + \phi_{yy} = 0$ by finding solution $\phi$ for which $\phi(z) = 0$ for each $z \in C_1$ and $\phi(z) = 10$ for each $z \in C_2$. Also, calculate the value of your solution at $z = 0$.

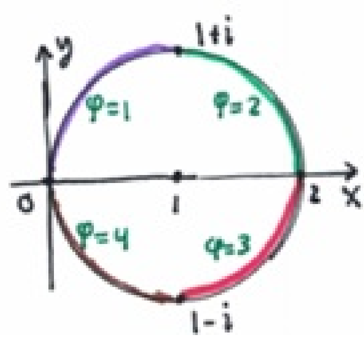
**Problem 100** Solve Laplace's equation on the unit-disk subject to the boundary condition $\phi(z) = 1$ for $|z| = 1$ with $\text{Re}(z) > 0$ and $\phi(z) = -1$ for $|z| = 1$ with $\text{Re}(z) < 0$. In other words, find the solution of Laplace's equation with value 1 on the right half unit-circle and value $-1$ on the left half unit-circle.

**Problem 101** Let $R$ be the annular region with center $1+i$ which is bounded by two circles. The inner circle has points 1 and $i$. The real and imaginary axes are tangent to the inner circle. The outside circle has the points $1-i$ and $1+i$. Solve Laplace's equation with boundary conditions $\phi = 10$ on the outside circle and $\phi = 0$ on the insider circle.

**Problem 102** Solve Laplace's Equation on the lens-shaped region bounded by radius one circles centered at $i$ and 1. You're given that $\phi = 0$ on the circle centered at $i$ and $\phi = 10$ on the circle centered at 1.



**Problem 103** Solve Laplace's Equation on the region below by exploiting the conformal mapping technique to the upper-half plane where there is a known solution for Laplace's equation with finite values specified on the real axis.




**Problem 104** Find the Fourier transform and verify the inversion formula for $F(t) = \frac{1}{t^2+9}$.

**Problem 105** This problem, borrowed from Saff and Snider, shows how Fourier transforms can be used to solve PDEs. You can assume differentiation commutes with integration here.

If $f(x)$ is a given function whose Fourier transform is $G(\omega)$, show that the function

$$u(x,t) = \int_{-\infty}^{\infty} G(\omega) e^{i\omega x} \cos \omega t \, d\omega$$

solves the partial differential equation

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\partial^2 u(x,t)}{\partial t^2}$$

and the initial conditions

$$u(x,0) = f(x)$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$

for $t > 0$ and $-\infty < x < \infty$. [These equations govern the motion of an infinite taut string initially displaced to the configuration $u = f(x)$ and released at $t = 0$.

**Problem 106** Let $a(n)$ be formed by the digits of $\pi$ in the sense that $a(n) = 0$ for all $n < 0$ and $a(0) = 3, a(1) = 1, a(2) = 4, a(3) = 1, \ldots$. Determine the domain of definition for the $z$-transform.

**Problem 107** Let $A(z)$ be the $z$-transform of the sequence $\{a(n) \mid -\infty < n < \infty\}$ in the annulus $a < |z| < b$. Show that the $z$-transform of the shifted sequence $\{b(n) = a(n+1) \mid -\infty < n < \infty\}$ us given by $zA(z)$. Also, conjecture what the $z$-transform for $\{c(n) = a(n+N) \mid -\infty < n < \infty\}$ is for any $N \in \mathbb{Z}$.

**Problem 108** Find the Hilbert transform of $\dfrac{a}{a^2+x^2}$ where $a > 0$.