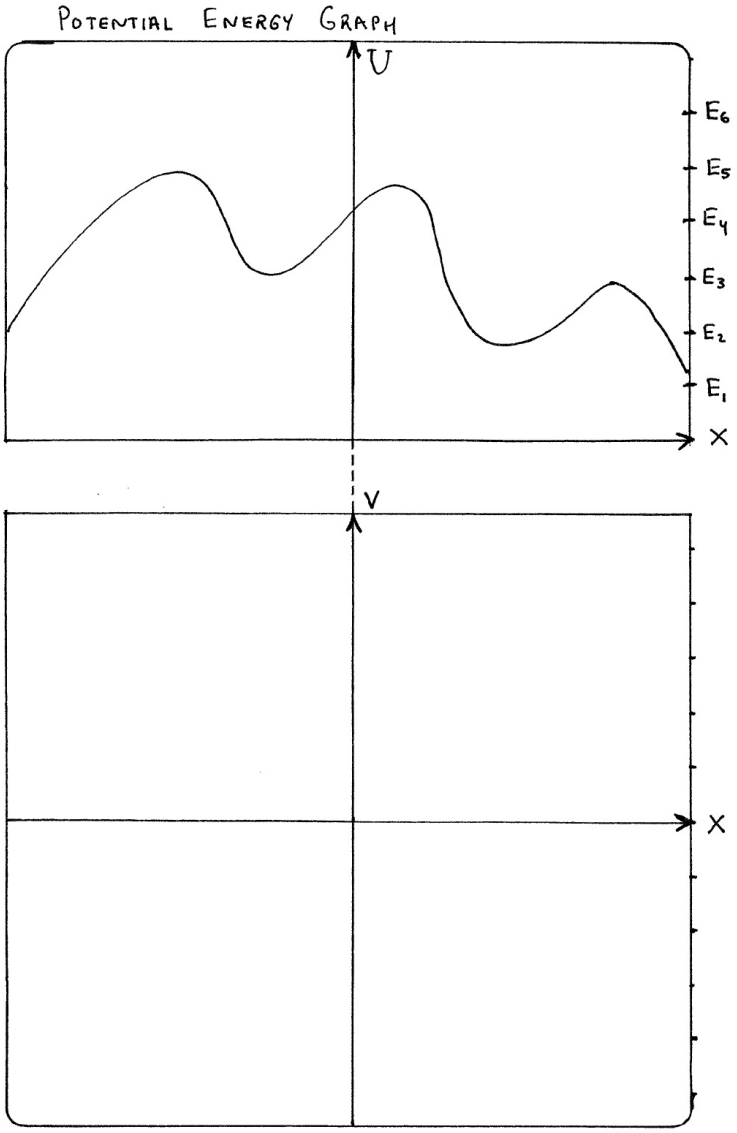
Name: (please print name as it appears in Canvas here →) .

MATH 334:	SAM 12: LAPLACE TRANSFORMS [30PTS]

Box or underline your answer. Work to be shown on separate, single-sided, normal sized paper with no fuzzies. Thanks!

Problem 72 (2pts) Plot the phase plane (or Poincare plot) given the potential energy plot below. For each energy $E_1, E_2, \ldots, E_6$ graph the corresponding trajectories below. Use a couple different colors so your work is easy to follow. Be neat. If no motion is possible then explain why.



Problem 73 (1pts) Use the Laplace transforms to solve the integral equation $f(t) + \int_0^t f(\tau)\, d\tau = 1$.

Problem 74 (1pts) Calculate $\mathcal{L}\{2t^2 e^{-t}\}(s)$

Problem 75 (1pts) Calculate $\mathcal{L}\{\sin(2t)\cos(3t)\}(s)$

Problem 76 (1pts) Calculate $\mathcal{L}\{te^t \sin(2t+3)\}(s)$

Problem 77 (4pts) Calculate the inverse Laplace transforms below:

(a.) $\mathcal{L}^{-1}\left\{\dfrac{6}{(s-1)^4}\right\}(t)$

(b.) $\mathcal{L}^{-1}\left\{\dfrac{2}{s^2+4}\right\}(t)$

(c.) $\mathcal{L}^{-1}\left\{\dfrac{s+1}{s^2+2s+10}\right\}(t)$

(d.) $\mathcal{L}^{-1}\left\{\dfrac{1}{s^5}\right\}(t)$

Problem 78 (1pts) Let $F(s) = \dfrac{3s-15}{2s^2-4s+10}$. Calculate $f = \mathcal{L}^{-1}\{F\}$.

Problem 79 (1pts) Let $F(s) = \dfrac{6s^2-13s+2}{s(s-1)(s-6)}$. Calculate $f = \mathcal{L}^{-1}\{F\}$.

Problem 80 (1pts) Let $F(s) = \dfrac{7s^3-2s^2-3s+6}{s^3(s-2)}$. Calculate $f = \mathcal{L}^{-1}\{F\}$.

Problem 81 (1pts) Let $F(s) = \tan^{-1}\left(\dfrac{1}{s}\right)$. Calculate $f = \mathcal{L}^{-1}\{F\}$.

Problem 82 (1pts) Solve $y'' + 6y' + 5y = 12e^t$ with $y(0) = -1$ and $y'(0) = 7$ via the Laplace transform technique.

Problem 83 (2pts) Calculate the Laplace transforms of the following functions

(a.) $f(t) = \sin(t)\cos(2t) + \sin^2(3t)$

(b.) $f(t) = e^t u(t-3) + \sin(t)u(t-6)$

Problem 84 (2pts) Solve $y'' + 4y' + 4y = u(t-\pi) - u(t-2\pi)$ with $y(0) = 0$ and $y'(0) = 0$ via the Laplace transform technique.

Problem 85 (2pts) Solve $y'' + 5y' + 6y = g(t)$ where $g(t) = \begin{cases} 0, & 0 < t < 1 \\ t, & 1 < t < 5 \\ 1, & 5 < t \end{cases}$

given $y(0) = 0$ and $y'(0) = 2$.

Problem 86 (2pts) Let $F(s) = \dfrac{e^{-\pi s}s}{s^2 + 6s + 13}$. Calculate the inverse Laplace transform of $F(s)$.

Problem 87 (2pts) Solve $w'' + w = \delta(t-\pi)$ where $w(0) = 0$ and $w'(0) = 0$.

Problem 88 (2pts) Solve $y'' + y = 4\delta(t-2) + t^2$ given $y(0) = 0$ and $y'(0) = 2$.

Problem 89 (1pts) A hammer hits a spring mass system at time $t = \pi/2$ and thus Newton's Second Law gives

$$\frac{d^2x}{dt^2} + 9x = -3\delta(t - \pi/2)$$

with $x(0) = 1$ and $x'(0) = 0$ since the spring is initially stretched to 1-unit and released from rest. Calculate the equation of motion and explain what happens after the hammer hits the spring at time $t = \pi/2$.

Problem 90 (1pts) Use convolution to find the inverse Laplace transform of $\dfrac{1}{s^2(s-a)}$ for $a \neq 0$.

Problem 91 (1pts) Find an integral solution of $y'' + y = g$ via Laplace transforms and convolution. You may assume g is an integrable function of time t.