

Name: (please print name as it appears in Canvas here →) _____.

MATH 334:	SAM 14: BVP, FOURIER SERIES AND PDES, LAGRANGIANS [25PTS]
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Box or underline your answer. Work to be shown on separate, single-sided, normal sized paper with no fuzzies. Thanks!

Problem 104 (3pts) Find all nontrivial solutions to the following Boundary Value Problems (BVPs) on the given interval I . For each case, determine what values of K permit a family of nontrivial solutions. Use $n \in \mathbb{N}$ to index the solutions as appropriate.

(a.) $y'' + Ky = 0$ with $y(0) = y(\pi) = 0$, $I = (0, \pi)$

(b.) $y'' + Ky = 0$ with $y'(0) = y'(1) = 0$, $I = (0, 1)$

(c.) $y'' + Ky = 0$ with $y'(0) = y(2) = 0$, $I = (0, 2)$

Problem 105 (2pt) Find the Fourier series which represents $f(x) = x^2$ on $-\pi < x < \pi$. Use your result to calculate the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ and $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$

Problem 106 (2pt) Find the Fourier sine series which represents $f(x) = x - x^2$ for $0 < x < 1$.

Problem 107 (2pt) Find the Fourier cosine series which represents $f(x) = \pi - x$ for $0 < x < \pi$.

Problem 108 (1pts) Consider separation of variables for the PDE given below

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = 0$$

given by the proposed product $u(r, \theta) = R(r)T(\theta)$. Show this proposal yields

$$T''(\theta) + \lambda T(\theta) = 0$$

where λ is a constant and find related ODE for R which also involves λ .

Problem 109 (4pts) Solve the heat equation $\frac{\partial u}{\partial t} = 3 \frac{\partial^2 u}{\partial x^2}$ on $0 < x < \pi$ for $t > 0$ given the boundary conditions $\frac{\partial u}{\partial x}(0, t) = \frac{\partial u}{\partial x}(\pi, t) = 0$ for $t > 0$ and initial condition $u(x, 0) = x$ for $0 < x < \pi$.

Problem 110 (5pts) Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ for $0 < x < 1$, $t > 0$ given the boundary conditions $u(0, t) = u(1, t) = 0$ for $t > 0$ and $u(x, 0) = x(1-x)$ and $\frac{\partial u}{\partial t}(x, 0) = \sin(7\pi x)$ for $0 < x < 1$.

Problem 111 (5pts) Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ for $0 \leq x, y \leq \pi$ given the boundary conditions $u(0, y) = u(\pi, y) = 0$ for $0 \leq y \leq \pi$ and $u(x, 0) = f(x)$ and $u(x, \pi) = 0$ for $0 \leq x \leq \pi$. You can expect the solution to be formed in terms of appropriate integrals involving f based on Fourier analytic argumentation.

Problem 112 (1pts) Let $L(x, y, \dot{x}, \dot{y}) = \frac{m}{2}(\dot{x}^2 + \dot{y}^2) + \alpha x^2 - \beta y^2$. Find the Euler-Lagrange equations for x and y .

Problem 113 (1pts) In physics, polar coordinates are sometimes given by $x = s \cos \phi$ and $y = s \sin \phi$. The **kinetic energy** of a mass m undergoing two-dimensional motion is given by $T = \frac{m}{2}(\dot{x}^2 + \dot{y}^2)$. Derive the formula for kinetic energy of a mass m in polar coordinates.

Problem 114 (1pts) Find the equations of motion for the polar coordinates of a free particle of mass m . Identify any conserved quantities.

Problem 115 (1pts) In physics the spherical coordinate system is given by $x = r \cos \phi \sin \theta$, $y = r \sin \phi \sin \theta$ and $z = r \cos \theta$. The **kinetic energy** of a mass m in Cartesian coordinates is given by $T = \frac{m}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$. Derive the formula for kinetic energy of a mass m in spherical coordinates.

Problem 116 (1pts) Find the equations of motion for the spherical coordinates of a free particle. Identify any conserved quantities.

Problem 117 (1pts) Suppose that the net-force on a given mass m is a conservative force which depends only on the distance from the origin. In this case, $F = -\frac{dU}{dr}$ and we can express the Lagrangian $L = T - U$ using the formula you derived in the previous problem and the arbitrary potential $U = U(r)$. Derive the Euler Lagrange equations for r , θ and ϕ .