Name: (please print name as it appears in Canvas here →) .

Math 334: Surprise Attack Mission 3: Exact ODEs and Substitution [15pts]

Same instructions as with SAM 2. I intend to collect this at the start of class 9-2-26.

Problem 1 (1pt) Solve $\big(xy^2 + y\cos(3xy)\big)\,dx + \big(x^2y + x\cos(3xy) + 4y^3\big)\,dy = 0$.

Problem 2 (1pt) Solve $\dfrac{2xy^3 + 5x^4y}{3x^2y^2 + x^5} + \dfrac{dy}{dx} = 0$.

Problem 3 (2pt) Solve $\big(3x^2y + y^2\big)\,dx + \big(x^2y^2 - xy\big)\,dy = 0$. This is an inexact equation, however, an integrating factor of the form $I = x^Ay^B$ should prove helpful.

Problem 4 (1pt) Let $M = 1 + ye^{xy}$ and $N = 2y + xe^{xy}$

(a.) Let $\vec{G} = \langle M, N\rangle$. Find the potential for this conservative vector field,

(b.) Solve $M dx + N dy = 0$.

Problem 5 (2pt) Define the exterior derivative of $\omega = Pdx + Qdy$ by $d\omega = dP \wedge dx + dQ \wedge dy$ where dP and dQ denote the usual total differentials of P and Q respective. The wedge product satisfies the following algebra rules:

$$(a\,dx + b\,dy) \wedge (c\,dx + e\,dy) = (ae - bc)dx \wedge dy$$

in other words, $dx \wedge dx = 0$ and $dy \wedge dy = 0$ and $dx \wedge dy = -dy \wedge dx$. With these rules we can show that if $Pdx + Qdy = 0$ is an exact equation then $\omega = Pdx + Qdy$ implies $d\omega = 0$ (this makes ω a **closed** differential form).

(a.) Let $\omega = (x + y^2)\,dx + (x^2 - \cos y)\,dy$. Calculate $d\omega$ and determine if $\omega = 0$ is an exact equation

(b.) Let $\omega = (2x\sinh(x^2 + y) + x^3)\,dx + (\sinh(x^2 + y) + 3) + \sin(y))\,dy$. Calculate $d\omega$ and determine if $\omega = 0$ is an exact equation.

(c.) Let $\omega = (3\theta^2\beta + 1)\,d\theta + (\theta^3 + \beta^2)\,d\beta$. Calculate $d\omega$ and determine if $\omega = 0$ is an exact equation.

Remark: notice I did **not** ask you to solve the DEqns in the above problem.

Problem 6 (8pt) Solve the following ODEs by making a wise substitution:

(a.) $\dfrac{dy}{dx} = (x + y + 1)^2$

(b.) $\dfrac{dy}{dx} = \dfrac{y - x}{y + x}$

(c.) $x\dfrac{dy}{dx} - (1 + x)y = xy^2$

(d.) $\big[2x(x^2 + y^2) + y\big]\,dx + \big[2y(x^2 + y^2) - x\big]\,dy = 0$